H.W. Alphinstone from I.6.211.

EXPERIMENTS ON COLOUR,

AS PERCEIVED BY THE EYE,

WITH

REMARKS ON COLOUR-BLINDNESS.

BY

JAMES CLERK MAXWELL, B.A.,

TRINITY COLLEGE, CAMBRIDGE.

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XVIII.—Experiments on Colour, as perceived by the Eye, with remarks on Colour-Blindness. By James Clerk Maxwell, B.A., Trinity College, Cambridge. Communicated by Dr Gregory. (With a Plate.)

(Read 19th March 1855.)

The object of the following communication is to describe a method by which every variety of visible colour may be exhibited to the eye in such a form as to admit of accurate comparison; to show how experiments so made may be registered numerically; and to deduce from these numerical results certain laws of vision.

The different tints are produced by means of a combination of discs of paper, painted with the pigments commonly used in the arts, and arranged round an axis, so that a sector of any required angular magnitude of each colour may be exposed. When this system of discs is set in rapid rotation, the sectors of the different colours become indistinguishable, and the whole appears of one uniform tint. The resultant tints of two different combinations of colours may be compared by using a second set of discs of a smaller size, and placing these over the centre of the first set, so as to leave the outer portion of the larger discs exposed. The resultant tint of the first combination will then appear in a ring round that of the second, and may be very carefully compared with it.

The form in which the experiment is most manageable is that of the common top. An axis, of which the lower extremity is conical, carries a circular plate, which serves as a support for the discs of coloured paper. The circumference of this plate is divided into 100 equal parts, for the purpose of ascertaining the proportions of the different colours which form the combination. When the discs have been properly arranged, the upper part of the axis is screwed down, so as to prevent any alteration in the proportions of the colours.

The instrument used in the first series of experiments (at Cambridge, in November 1854) was constructed by myself, with coloured papers procured from Mr D. R. Hay. The experiments made in the present year were with the improved top made by Mr J. M. Bryson, Edinburgh, and coloured papers prepared by Mr T. Purdie, with the unmixed pigments used in the arts. A number of Mr Bryson's tops, with Mr Purdie's coloured papers has been prepared, so as to afford different observers the means of testing and comparing results independently obtained.

The colours used for Mr Purdie's papers were-

Vermilion, .		V	Ultramarine, .	•	U	Emerald Green, .	EG
Carmine, .		C	Prussian Blue,	-	PΒ	Brunswick Green, .	\mathbf{BG}
Red Lead, .		RL	Verditer Blue,		$\mathbf{v}_{\mathbf{B}}$	Mixture of Ultramarine	
Orange Orpiment,		00				and Chrome, .	\mathbf{uc}
Orange Chrome,		oc					
Chrome Yellow,	•	\mathbf{CY}					
Gamboge, .		\mathbf{Gam}					
Pale Chrome, .		\mathbf{PC}					
			Ivory Black, .	-	Bk		
			Snow White, .		sw		
			White Paper (Pirie	, Aber	deen).		

The colours in the first column are reds, oranges, and yellows; those in the second, blues; and those in the third, greens. Vermilion, ultramarine, and emerald green, seem the best colours to adopt in referring the rest to a uniform standard. They are therefore put at the head of the list, as types of three convenient divisions of colour, red, blue, and green.

It may be asked, why some variety of yellow was not chosen in place of green, which is commonly placed among the secondary colours, while yellow ranks as a primary? The reason for this deviation from the received system is, that the colours on the discs do not represent primary colours at all, but are simply specimens of different kinds of paint, and the choice of these was determined solely by the power of forming the requisite variety of combinations. Now, if red, blue, and yellow, had been adopted, there would have been a difficulty in forming green by any compound of blue and yellow, while the yellow formed by vermilion and emerald green is tolerably distinct. This will be more clearly perceived after the experiments have been discussed, by referring to the diagram.

As an example of the method of experimenting, let us endeavour to form a neutral gray by the combination of vermilion, ultramarine, and emerald green. The most perfect results are obtained by two persons acting in concert, when the operator arranges the colours and spins the top, leaving the eye of the observer free from the distracting effect of the bright colours of the papers when at rest.

After placing discs of these three colours on the circular plate of the top, and smaller discs of white and black above them, the operator must spin the top, and demand the opinion of the observer respecting the relation of the outer ring to the inner circle. He will be told that the outer circle is too red, too blue, or too green, as the case may be, and that the inner one is too light or too dark, as compared with the outer. The arrangement must then be changed, so as to render the resultant tint of the outer and inner circles more nearly alike. Sometimes the observer will see the inner circle tinted with the complementary colour of the outer one. In this case the operator must interpret the observation with respect to the outer circle, as the inner circle contains only black and white.

By a little experience the operator will learn how to put his questions, and

how to interpret their answers. The observer should not look at the coloured papers, nor be told the proportions of the colours during the experiments. When these adjustments have been properly made, the resultant tints of the outer and inner circles ought to be perfectly indistinguishable, when the top has a sufficient velocity of rotation. The number of divisions occupied by the different colours must then be read off on the edge of the plate, and registered in the form of an equation. Thus, in the preceding experiment we have vermilion, ultramarine, and emerald green outside, and black and white inside. The numbers, as given by an experiment on the 6th March 1855, in daylight without sun, are—

$$\cdot 37 \text{ V} + \cdot 27 \text{ U} + \cdot 36 \text{ EG} = \cdot 28 \text{ SW} + \cdot 72 \text{ Bk}$$
 . (1).

The method of treating these equations will be given when we come to the theoretical view of the subject.

In this way we have formed a neutral gray by the combination of the three standard colours. We may also form neutral grays of different intensities by the combination of vermilion and ultramarine with the other greens, and thus obtain the quantities of each necessary to neutralize a given quantity of the proposed green. By substituting for each standard colour in succession one of the colours which stand under it, we may obtain equations, each of which contains two standard colours, and one of the remaining colours.

Thus, in the case of pale chrome, we have, from the same set of experiments,

$$\cdot 34 \text{ PC} + \cdot 55 \text{ U} + \cdot 12 \text{ EG} = \cdot 37 \text{ SW} + \cdot 63 \text{ Bk}$$
 . . . (2).

We may also make experiments in which the resulting tint is not a neutral gray, but a decided colour. Thus we may combine ultramarine, pale chrome, and black, so as to produce a tint identical with that of a compound of vermilion and emerald green. Experiments of this sort are more difficult, both from the inability of the observer to express the difference which he detects in two tints which have, perhaps, the same hue and intensity, but differ in purity; and also from the complementary colours which are produced in the eye after gazing too long at the colours to be compared.

The best method of arriving at a result in the case before us, is to render the hue of the red and green combination something like that of the yellow, to reduce the purity of the yellow by the admixture of blue, and to diminish its intensity by the addition of black. These operations must be repeated and adjusted, till the two tints are not merely varieties of the same colour, but absolutely the same. An experiment made 5th March gives—

$$\cdot 39 \text{ PC} + \cdot 21 \text{ U} + \cdot 40 \text{ Bk} = \cdot 59 \text{ V} + \cdot 41 \text{ EG}$$
 . (3).

That these experiments are really evidence relating to the constitution of the eye, and not mere comparisons of two things which are in themselves identical, may be shown by observing these resultant tints through coloured glasses, or by using

gas-light instead of day-light. The tints which before appeared identical will now be manifestly different, and will require alteration, to reduce them to equality.

Thus, in the case of carmine, we have by day-light,

while by gas-light (Edinburgh)

which shows that the yellowing effect of the gas-light tells more on the white than on the combination of colours. If we examine the two resulting tints which appeared identical in experiment (3), observing the whirling discs through a blue glass, the combination of yellow, blue, and black, appears redder than the other, while through a yellow glass, the red and green mixture appears redder. So also a red glass makes the first side of the equation too dark, and a green glass makes it too light.

The apparent identity of the tints in these experiments is therefore not real, but a consequence of a determinate constitution of the eye, and hence arises the importance of the results, as indicating the laws of human vision.

The first result which is worthy of notice is, that the equations, as observed by different persons of ordinary vision, agree in a remarkable manner. If care be taken to secure the same kind of light in all the experiments, the equations, as determined by two independent observers, will seldom show a difference of more than three divisions in any part of the equation containing the bright standard colours. As the duller colours are less active in changing the resultant tint, their true proportions cannot be so well ascertained. The accuracy of vision of each observer may be tested by repeating the same experiment at different times, and comparing the equations so found.

Experiments of this kind, made at Cambridge in November 1854, show that of ten observers, the best were accurate to within 1½ division, and agreed within 1 division of the mean of all; and the worst contradicted themselves to the extent of 6 degrees, but still were never more than 4 or 5 from the mean of all the observations.

We are thus led to conclude—

- 1st, That the human eye is capable of estimating the likeness of colours with a precision which in some cases is very great.
- 2d, That the judgment thus formed is determined, not by the real identity of the colours, but by a cause residing in the eye of the observer.
- 3d, That the eyes of different observers vary in accuracy, but agree with each other so nearly as to leave no doubt that the law of colour-vision is identical for all ordinary eyes.

Investigation of the Law of the Perception of Colour.

Before proceeding to the deduction of the elementary laws of the perception of colour from the numerical results previously obtained, it will be desirable to point out some general features of the experiments which indicate the form which these laws must assume.

Returning to experiment (1), in which a neutral gray was produced from red, blue, and green, we may observe, that, while the adjustments were incomplete, the difference of the tints could be detected only by one circle appearing more red, more green, or more blue than the other, or by being lighter or darker, that is, having an excess or defect of all the three colours together. Hence it appears that the nature of a colour may be considered as dependent on three things, as, for instance, redness, blueness, and greenness. This is confirmed by the fact, that any tint may be imitated by mixing red, blue, and green alone, provided that tint does not exceed a certain brilliancy.

Another way of showing that colour depends on three things is, by considering how two tints, say two lilacs, may differ. In the first place, one may be lighter or darker than the other, that is, the tints may differ in shade. Secondly, one may be more blue or more red than the other, that is, they may differ in hue. Thirdly, one may be more or less decided in its colour; it may vary from purity on the one hand, to neutrality on the other. This is sometimes expressed by saying that they may differ in tint.

Thus, in shade, hue, and tint, we have another mode of reducing the elements of colour to three. It will be shown that these two methods of considering colour may be deduced one from the other, and are capable of exact numerical comparison.

On a Graphical Method of Exhibiting the Relations of Colours.

The method which exhibits to the eye most clearly the results of this theory of the three elements of colour, is that which supposes each colour to be represented by a point in space, whose distances from three co-ordinate planes are proportional to the three elements of colour. But as any method by which the operations are confined to a plane is preferable to one requiring space of three dimensions, we shall only consider for the present that which has been adopted for convenience, founded on Newton's Circle of Colours and Mayer and Young's Triangle.

Vermilion, ultramarine, and emerald green, being taken (for convenience) as standard colours, are conceived to be represented by three points, taken (for convenience) at the angles of an equilateral triangle. Any colour compounded of these three is to be represented by a point found by conceiving masses propor-

tional to the several components of the colour placed at their respective angular points, and taking the centre of gravity of the three masses. In this way, each colour will indicate by its position the proportions of the elements of which it is composed. The total intensity of the colour is to be measured by the whole number of divisions of V, U, and EG, of which it is composed. This may be indicated by a number or coefficient appended to the name of the colour, by which the number of divisions it occupies must be multiplied to obtain its mass in calculating the results of new combinations.

This will be best explained by an example on the diagram (No. 1). We have, by experiment (1),

$$\cdot 37 \text{ V} + \cdot 27 \text{ U} + \cdot 36 \text{ EG} = \cdot 28 \text{ SW} + \cdot 72 \text{ Bk}$$

To find the position of the resultant neutral tint, we must conceive a mass of $\cdot 37$ at V, of $\cdot 27$ at U, and of $\cdot 36$ at EG, and find the centre of gravity. This may be done by taking the line UV, and dividing it in the proportion of $\cdot 37$ to $\cdot 27$ at the point α , where

$$\alpha \mathbf{V} : \alpha \mathbf{U} : : \cdot 27 : \cdot 37$$

Then, joining a with EG, divide the joining line in W in the proportion of :36 to (37+27), W will be the position of the neutral tint required, which is not white, but 0.28 of white, diluted with 0.72 of black, which has hardly any effect whatever, except in decreasing the amount of the other colour. The total intensity of our white paper will be represented by $\frac{1}{0.28}=3.57$; so that, whenever white enters into an equation, the number of divisions must be multiplied by the coefficient 3.57 before any true results can be obtained.

We may take, as the next example, the method of representing the relation of pale chrome to the standard colours on our diagram, by making use of experiment (2), in which pale chrome, ultramarine, and emerald green, produced a neutral gray. The resulting equation was

$$\cdot 33 \text{ PC} + \cdot 55 \text{ U} + \cdot 12 \text{ EG} = \cdot 37 \text{ SW} + \cdot 63 \text{ Bk}$$
 . (2).

In order to obtain the total intensity of white, we must multiply the number of divisions, 37, by the proper coefficient, which is 3.57. The result is 1.32, which therefore measures the total intensity on both sides of the equation.

Subtracting the intensity of .55 U + .12 EG, or .67 from 1.32, we obtain .65 as the corrected value of .32 PC. It will be convenient to use these corrected values of the different colours, taking care to distinguish them by small initials instead of capitals.

Equation (2) then becomes

$$\cdot 65 \text{ pc} + \cdot 55 \text{ U} + \cdot 12 \text{ EG} = 1 \cdot 32 \text{ w}$$

Hence pc must be situated at a point such that w is the centre of gravity of $\cdot 65$ pc + $\cdot 55$ U + $\cdot 12$ EG.

To find it, we begin by determining β the centre of gravity of 55 U + 12 EG, then, joining β w, the point we are seeking must lie at a certain distance on the other side of w from c. This distance may be found from the proportion,

$$\cdot 65 : (\cdot 55 + \cdot 12) :: \overline{\beta w} : \overline{w pc}$$

which determines the position of pc. The proper coefficient, by which the observed values of PC must be corrected is $\frac{65}{331}$ or 1.97.

We have thus determined the position and coefficient of a colour by a single experiment, in which it was made to produce a neutral tint along with two of the standard colours. As this may be done with every possible colour, the method is applicable wherever we can obtain a disc of the proposed colour. In this way the diagram (No. 1) has been laid down from observations made in daylight, by a good eye of the ordinary type.

It has been observed that experiments, in which the resultant tint is neutral, are more accurate than those in which the resulting tint has a decided colour, as in experiment (3), owing to the effects of accidental colours produced in the eye in the latter case. These experiments, however, may be repeated till a very good mean result has been obtained.

But since the elements of every colour have been already fixed by our previous observations and calculations, the agreement of these results with those calculated from the diagram forms a test of the correctness of our method.

By experiment (No. 3), made at the same time with (1) and (2), we have

Now, joining U with p c, and V with EG, the only common point is that at which they cross, namely γ .

Measuring the parts of the line VEG, we find them in the proportion of

Similarly, the line \overline{Upc} is divided in the proportion

·78 p c and ·22
$$U=1.00 \gamma$$

But 78 pc must be divided by 1.97, to reduce it to PC, as was previously explained. The result of calculation is, therefore,

$$\cdot 39 \text{ PC} + \cdot 22 \text{ U} + \cdot 39 \text{ Bk} = \cdot 58 \text{ V} + \cdot 42 \text{ EG}$$

the black being introduced simply to fill up the circle.

This result differs very little from that of experiment (3), and it must be recollected that these are single experiments, made independently of theory, and chosen at random. Experiments made at Cambridge, with all the combinations of five colours, show that theory agrees with calculation always within 0.012 of the whole, and sometimes within 0.002. By the repetition of these experiments at the numerous opportunities which present themselves, the accuracy of the results may be rendered still greater. As it is, I am not aware that the judgments of the human eye with respect to colour have been supposed capable of so severe a test.

Further consideration of the Diagram of Colours.

We have seen how the composition of any tint, in terms of our three standard colours, determines its position on the diagram and its proper coefficient. In the same way, the result of mixing any other colours, situated at other points of the diagram, is to be found by taking the centre of gravity of their reduced masses, as was done in the last calculation (experiment 3).

We have now to turn our attention to the general aspect of the diagram.

The standard colours, V, U, and EG, occupy the angles of an equilateral triangle, and the rest are arranged in the order in which they participate in red, blue, and green, the neutral tint being at the point w within the triangle. If we now draw lines through w to the different colours ranged round it, we shall find that, if we pass from one line to another in the order in which they lie from red to green, and through blue back again to red, the order will be—

		Coefficient.			Coefficient,
Carmine;		0.4	Pale Chrome .		2.0
Vermilion,		1.0	Mixed Green (UC),		0.4
Red Lead, .		1.3	Brunswick Green,		0.2
Orange Orpiment,		1.0	Emerald Green,		1.0
Orange Chrome,		1.6	Verditer Blue, .		8.0
Chrome Yellow,		1.5	Prussian Blue, .		0.1
Gamboge,		1.8	Ultramarine, .		1.0

It may be easily seen that this arrangement of the colours corresponds to that of the prismatic spectrum; the only difference being that the spectrum is deficient in those fine purples which lie between ultramarine and vermilion, and which are easily produced by mixture. The experiments necessary for determining the exact relation of this list to the lines in the spectrum are not yet completed.

If we examine the colours represented by different points in one of these lines through w, we shall find the purest and most decided colours at its outer extremity, and the faint tints approaching to neutrality nearer to w.

If we also study the coefficients attached to each colour, we shall find that the brighter and more luminous colours have higher numbers for their coefficients than those which are dark.

In this way, the qualities which we have already distinguished as hue, tint, and shade, are represented on the diagram by angular position with respect to w, distance from w, and coefficient; and the relation between the two methods of reducing the elements of colour to three becomes a matter of geometry.

Theory of the Perception of Colour.

Opticians have long been divided on this point; those who trusted to popular notions and their own impressions adopting some theory of three primary colours, while those who studied the phenomena of light itself proved that no such theory could explain the constitution of the spectrum. Newton, who was the first to demonstrate the actual existence of a series of kinds of light, countless in number, yet all perfectly distinct, was also the first to propound a method of calculating the effect of the mixture of various coloured light; and this method was substantially the same as that which we have just verified. It is true, that the directions which he gives for the construction of his circle of colours are somewhat arbitrary, being probably only intended as an indication of the general nature of the method, but the method itself is mathematically reducible to the theory of three elements of the colour-sensation.*

Young, who made the next great step in the establishment of the theory of light, seems also to have been the first to follow out the necessary consequences of Newton's suggestion on the mixture of colours. He saw that, since this triplicity has no foundation in the theory of light, its cause must be looked for in the constitution of the eye; and, by one of those bold assumptions which sometimes express the result of speculation better than any cautious trains of reasoning, he attributed it to the existence of three distinct modes of sensation in the retina, each of which he supposed to be produced in different degrees by the different rays. These three elementary effects, according to his view, correspond to the three sensations of red, green, and violet, and would separately convey to the sensorium the sensation of a red, a green, and a violet picture; so that by the superposition of these pictures, the actual variegated world is represented.†

In order fully to understand Young's theory, the function which he attributes to each system of nerves must be carefully borne in mind. Each nerve acts, not, as some have thought, by conveying to the mind the knowledge of the length of an undulation of light, or of its periodic time, but simply by being more or less affected by the rays which fall on it. The sensation of each elementary nerve is capable only of increase and diminution, and of no other change. We must also observe, that the nerves corresponding to the red sensation are affected chiefly by the red rays, but in some degree also by those of every other part of the spectrum; just as red glass transmits red rays freely, but also suffers those of other colours to pass in smaller quantity.

This theory of colour may be illustrated by a supposed case taken from

^{*} See Note III. For a confirmation of Newton's analysis of Light, see Helmholtz. Pogg. Ann. 1852; and Phil. Mag. 1852, Part II.

[†] Young's Lectures, p. 345, Kelland's Edition. See also Helmholtz's statement of Young's Theory, in his Paper referred to in Note I.; and Herschel's Light, Art. 518.

the art of photography. Let it be required to ascertain the colours of a landscape, by means of impressions taken on a preparation equally sensitive to rays of every colour.

Let a plate of red glass be placed before the camera, and an impression taken. The positive of this will be transparent wherever the red light has been abundant in the landscape, and opaque where it has been wanting. Let it now be put in a magic lantern, along with the red glass, and a red picture will be thrown on the screen.

Let this operation be repeated with a green and a violet glass, and, by means of three magic lanterns, let the three images be superimposed on the screen. The colour of any point on the screen will then depend on that of the corresponding point of the landscape; and, by properly adjusting the intensities of the lights, &c., a complete copy of the landscape, as far as visible colour is concerned, will be thrown on the screen. The only apparent difference will be, that the copy will be more subdued, or less pure in tint, than the original. Here, however, we have the process performed twice—first on the screen, and then on the retina.

This illustration will show how the functions which Young attributes to the three systems of nerves may be imitated by optical apparatus. It is therefore unnecessary to search for any direct connection between the lengths of the undulations of the various rays of light and the sensations as felt by us, as the three-fold partition of the properties of light may be effected by physical means. The remarkable correspondence between the results of experiments on different individuals would indicate some anatomical contrivance identical in all. As there is little hope of detecting it by dissection, we may be content at present with any subsidiary evidence which we may possess. Such evidence is furnished by those individuals who have the defect of vision which was described by Dalton, and which is a variety of that which Dr G. Wilson has lately investigated, under the name of Colour-Blindness.

Testimony of the Colour-blind with respect to Colour.

Dr George Wilson has described a great number of cases of colour-blindness, some of which involve a general indistinctness in the appreciation of colour, while in others, the errors of judgment are plainly more numerous in those colours which approach to red and green, than among those which approach to blue and yellow. In these more definite cases of colour-blindness, the phenomena can be tolerably well accounted for by the hypothesis of an insensibility to red light; and this is, to a certain extent, confirmed by the fact, that red objects appear to these eyes decidedly more obscure than to ordinary eyes. But by experiments made with the pure spectrum, it appears that though the red appears much more obscure than other colours, it is not wholly invisible, and, what is more curious, resembles the green more than any other colour. The spectrum to them

appears faintly luminous in the red; bright yellow from orange to yellow, bright but not coloured from yellow-green to blue, and then strongly coloured in the extreme blue and violet, after which it seems to approach the neutral obscure tint of the red. It is not easy to see why an insensibility to red rays should deprive the green rays, which have no optical connection with them, of their distinctive appearance. The phenomena seem rather to lead to the conclusion that it is the red sensation which is wanting, that is, that supposed system of nerves which is affected in various degrees by all light, but chiefly by red. We have fortunately the means of testing this hypothesis by numerical results.

Of the subjects of my experiments at Cambridge, four were decided cases of colour-blindness. Of these two, namely, Mr R. and Mr S., were not sufficiently critical in their observations to afford any results consistent within 10 divisions of the colour-top. The remaining two, Mr N. and Mr X., were as consistent in their observations as any persons of ordinary vision can be, while the results showed all the more clearly how completely their sensations must differ from ours.

The method of experimenting was the same as that adopted with ordinary eyes, except that in these cases the operator can hardly influence the result by yielding to his own impressions, as he has no perception whatever of the similarity of the two tints as seen by the observer. The questions which he must ask are two, Which circle appears most blue or yellow? Which appears lightest and which darkest? By means of the answers to these questions he must adjust the resulting tints to equality in these respects as it appears to the observer, and then ascertain that these tints now present no difference of colour whatever to his eyes. The equations thus obtained do not require five colours including black, but four only. For instance, the mean of several observations gives—

$$\cdot 19 \text{ G} + \cdot 05 \text{ B} + \cdot 76 \text{ Bk} = 100 \text{ R}$$
 (4).

[In these experiments R, B, G, Y, stand for red, blue, green, and yellow papers prepared by Mr D. R. Hay. I am not certain that they are identical with his standard colours, but I believe so. Their relation to vermilion, ultramarine, and emerald-green is given in diagram (1). Their relations to each other are very accurately given in diagram (2).

It appears, then, that the dark blue-green of the left side of the equation is equivalent to the full red of the right side.

Hence, if we divide the line BG in the proportion of 19 to 5 at the point β , and join R β , the tint at β will differ from that at R (to the colour-blind) only in being more brilliant in the proportion of 100 to 24, and all intermediate tints on the line R β will appear to them of the same hue, but of intermediate intensities.

Now, if we take a point D, so that RD is to $R\beta$ in the proportion of 24 to 100-24, or 76, the tint of D, if producible, should be invisible to the colour-blind. D, therefore, represents the pure sensation which is unknown to the colour-blind.

and the addition of this sensation to any others cannot alter it in their estimation. It is for them equivalent to black.

Hence, if we draw lines through D in different directions, the colours belonging to the line ought to differ only in intensity as seen by them, so that one of them may be reduced to the other by the addition of black only. If we draw DW and produce it, all colours on the upper side of DW will be varieties of blue, and those on the under side varieties of yellow, so that the line DW is a boundary line between their two kinds of colour, blue and yellow being the names by which they call them.

The accuracy of this theory will be evident from the comparison of the experiments which I had an opportunity of making on Mr N. and Mr X. with each other, and with measurements taken from the diagram No. 2, which was constructed from the observations of ordinary eyes only, the point D alone being ascertained from a series of observations by Mr N.

Taking the point γ , between R and B, it appears, by measurement of the lines R γ and B γ , that γ corresponds to

By measurement of W γ and D γ , and correction by means of the coefficient of W, and calling D black in the colour-blind language, γ corresponds to

$$105 \text{ W} + 895 \text{ Bk}$$

Therefore,

By measurement,
$$.93 R + .07 B = .105 W + .895 Bk$$

By observation N. & X. together, $.94 R + .06 B = .10 W + .90 Bk$
By X. alone, $.93 R + .07 B = .10 W + .90 Bk$ (5).

The agreement here is as near as can be expected.

By a similar calculation with respect to the point δ , between B and G,

We may also observe, that the line GD crosses RY. At the point of intersection we have—

Here observations are at variance, owing to the decided colours produced affecting the state of the retina, but the mean agrees well with calculation.

Drawing the line BY, we find that it cuts lines through D drawn to every

colour. Hence all colours appear to the colour-blind as if composed of blue and yellow. By measurement on the diagram, we find for red

For green we have in the same way—

For white—

Measured,
$$407 \text{ Y} + 593 \text{ B} = 326 \text{ W} + 674 \text{ Bk}$$

Observed by N., $40 \text{ Y} + 60 \text{ B} = 33 \text{ W} + 67 \text{ Bk}$
... X., $44 \text{ Y} + 56 \text{ B} = 33 \text{ W} + 67 \text{ Bk}$

The accuracy of these results shows that, whether the hypothesis of the want of one element out of three necessary to perfect vision be actually true or not, it affords a most trustworthy foundation on which to build a theory of colour-blindness, as it expresses completely the observed facts of the case. They also furnish us with a datum for our theory of perfect vision, namely, the point D, which points out the exact nature of the colour-sensation, which must be added to the colour-blind eye to render it perfect. I am not aware of any method of determining by a legitimate process the nature of the other two sensations, although Young's reasons for adopting something like green and violet appear to me worthy of attention.

The only remaining subject to which I would call the attention of the Society is the effect of coloured glasses on the colour-blind. Although they cannot distinguish reds and greens from varieties of gray, the transparency of red and green glasses for those kinds of light is very different. Hence, after finding a case such as that in equation (4), in which a red and a green appear identical, on looking through a red glass they see the red clearly and the green obscurely, while through a green glass the red appears dark and the green light.

By furnishing Mr X. with a red and a green glass, which he could distinguish only by their shape, I enabled him to make judgments in previously doubtful cases of colour with perfect certainty. I have since had a pair of spectacles constructed with one eye glass red and the other green. These Mr X. intends to use for a length of time, and he hopes to acquire the habit of discriminating red from green tints by their different effects on his two eyes. Though he can never acquire our sensation of red, he may then discern for himself what things are red, and the mental process may become so familiar to him as to act unconsciously like a new sense.

In one experiment, after looking at a bright light, with a red glass over one eye and a green over the other, the two tints in experiment (4) appeared to him altered, so that the outer circle was lighter according to one eye, and the inner according to the other. As far as I could ascertain, it appeared as if the eye which had used the red glass saw the red circle brightest. This result, which seems at variance with what might be expected, I have had no opportunity of verifying.

This paper is already longer than was originally intended. For further information I would refer the reader to Newton's Opticks, Book I. Part II., to Young's Lectures on Natural Philosophy, page 345, to Mr D. R. Hay's works on Colours, and to Professor Forbes on the Classification of Colours (Phil. Mag., March 1849).

The most remarkable paper on the subject is that of M. Helmholtz, in the Philosophical Magazine for 1852, in which he discusses the different theories of primary colours, and describes his method of mixing the colours of the spectrum. An examination of the results of M. Helmholtz with reference to the theory of three elements of colour, by Professor Grassmann, is translated in the Phil. Mag., April 1854.

References to authors on colour-blindness are given in Dr G. Wilson's papers on that subject. A valuable Letter of Sir J. F. W. Herschel to Dalton on his peculiarity of vision, is to be found in the Life of Dalton by Dr Henry.

I had intended to describe some experiments on the propriety of the method of mixing colours by rotation, which might serve as an extension of Mr Swan's experiments on instantaneous impressions on the eye. These, together with the explanation of some phenomena which seem to be at variance with the theory of vision here adopted, must be deferred for the present. On some future occasion, I hope to be able to connect these simple experiments on the colours of pigments with others in which the pure hues of the spectrum are used. I have already constructed a model of apparatus for this purpose, and the results obtained are sufficiently remarkable to encourage perseverance.

NOTE I.

On different Methods of Exhibiting the Mixtures of Colours.

(1.) Mechanical Mixture of Coloured Powders.

By grinding coloured powders together, the differently-coloured particles may be so intermingled that the eye cannot distinguish the colours of the separate powders, but receives the impression of a uniform tint, depending on the nature and proportions of the pigments used. In this way, Newton mixed the powders of orpiment, purple, bise, and viride æris, so as to form a gray, which, in sunlight, resembled white paper in the shade. (Newton's Opticks, Book I. Part II. Exp.

XV.) This method of mixture, besides being adopted by all painters, has been employed by optical writers as a means of obtaining numerical results. The specimens of such mixtures given by D. R. Hay in his works on Colour, and the experiments of Professor J. D. Forbes on the same subject, show the importance of the method as a means of classifying colours. There are two objections, however, to this method of exhibiting colours to the eye. When two powders of unequal fineness are mixed, the particles of the finer powder cover over those of the coarser, so as to produce more than their due effect in influencing the resultant tint. For instance, a small quantity of lamp-black, mixed with a large quantity of chalk, will produce a mixture which is nearly black. Although the powders generally used are not so different in this respect as lamp-black and chalk, the results of mixing given weights of any coloured powders must be greatly modified by the mode in which these powders have been prepared.

Again, the light which reaches the eye from the surface of the mixed powders consists partly of light which has fallen on one of the substances mixed without being modified by the other, and partly of light which, by repeated reflection or transmission has been acted on by both substances. The colour of these rays will not be a mixture of those of the substances, but will be the result of the absorption due to both substances successively. Thus, a mixture of yellow and blue produces a neutral tint tending towards red, but the remainder of white light, after passing through both, is green; and this green is generally sufficiently powerful to overpower the reddish gray due to the separate colours of the substances mixed. This curious result has been ably investigated by Professor Helmboltz of Königsberg, in his Memoir on the Theory of Compound Colours, a translation of which may be found in the Annals of Philosophy for 1852, Part 2.

(2.) Mixture of differently-coloured Beams of Light by Superposition on an Opaque Screen.

When we can obtain light of sufficient intensity, this method produces the most beautiful results. The best series of experiments of this kind are to be found in Newton's Opticks, Book I. Part II. The different arrangements for mixing the rays of the spectrum on a screen, as described by Newton, form a very complete system of combinations of lenses and prisms, by which almost every possible modification of coloured light may be produced. The principal objections to the use of this method are—(1.) The difficulty of obtaining a constant supply of uniformly intense light; (2.) The uncertainty of the effect of the position of the screen with respect to the incident beams and the eye of the observer; (3.) The possible change in the colour of the incident light due to the fluorescence of the substance of the screen. Professor Stokes has found that many substances, when illuminated by homogeneous light of one refrangibility, become themselves luminous, so as to emit light of lower refrangibility. This phenomenon must be carefully attended to when screens are used to exhibit light.

(3.) Union of Coloured Beams by a Prism so as to form one Beam.

The mode of viewing the beam of light directly, without first throwing it on a screen, wasnot much used by the older experimenters, but it possesses the advantage of saving much light, and admits of examining the rays before they have been stopped in any way. In Newton's 11th proposition of the 2d Book, an experiment is described, in which a beam is analysed by a prism, concentrated by a lens, and recombined by another prism, so as to form a beam of white light similar to the incident beam. By stopping the coloured rays at the lens, any proposed combination may be made to pass into the emergent beam, where it may be received directly by the eye, or on a screen, at pleasure

The experiments of Helmholtz on the colours of the spectrum were made with the ordinary apparatus for directly viewing the pure spectrum, two oblique slits crossing one another being employed to admit the light instead of one vertical slit. Two pure spectra were then seen crossing each other, and so exhibiting at once a large number of combinations. The proportions of these combinations were altered by varying the inclination of the slits to the plane of refraction, and in this way a number of very remarkable results were obtained,—for which see his Memoir, before referred to.

In experiments of the same kind made by myself in August 1852 (independently of M. Helmholtz), I used a combination of three moveable vertical slits to admit the light, instead of two cross slits, and observed the compound ray through a slit made in a screen on which the pure spectrum is formed. In this way a considerable field of view was filled with the mixed light, and might be compared with another part of the field illuminated by light proceeding from a second system of slits, placed below the first set. The general character of the results agreed with those of M. Helmholtz. The chief difficulties seemed to arise from the defects of the optical apparatus of my own eye, which rendered apparent the compound nature of the light, by analysing it as a prism or an ordinary lens would do, whenever the lights mixed differed much in refrangibility.

(4.) Union of two beams by means of a tra sparent surface, which reflects the first and transmits the second.

The simplest experiment of this kind is described by M. Helmholtz. He places two coloured wafers on a table, and then, taking a piece of transparent glass, he places it between them, so that the reflected image of one apparently coincides with the other as seen through the glass. The colours are thus mixed, and, by varying the angle of reflection, the relative intensities of the reflected and transmitted beams may be varied at pleasure.

In an instrument constructed by myself for photometrical purposes two reflecting plates were used. They were placed in a square tube, so as to polarize

the incident light, which entered through holes in the sides of the tubes, and was reflected in the direction of the axis. In this way two beams oppositely polarized were mixed, either of which could be coloured in any way by coloured glasses placed over the holes in the tube. By means of a Nicol's prism placed at the end of the tube, the relative intensities of the two colours as they entered the eye could be altered at pleasure.

(5.) Union of two coloured beams by means of a doubly-refracting Prism.

I am not aware that this method has been tried, although the opposite polarization of the emergent rays is favourable to the variation of the experiment.

(6.) Successive presentation of the different Colours to the Retina.

It has long been known, that light does not produce its full effect on the eye at once, and that the effect, when produced, remains visible for some after the light has ceased to act. In the case of the rotating disc, the various colours become indistinguishable, and the disc appears of a uniform tint, which is in some sense the resultant of the colours so blended. This method of combining colours has been used since the time of Newton, to exhibit the results of theory. The experiments of Professor J. D. Forbes, which I witnessed in 1849, first encouraged me to think that the laws of this kind of mixture might be discovered by special experiments. After repeating the well-known experiment in which a series of colours representing those of the spectrum are combined to form gray, Professor Forbes endeavoured to form a neutral tint, by the combination of three colours only. For this purpose, he combined the three so-called primary colours, red, blue, and yellow, but the resulting tint could not be rendered neutral by any combination of these colours; and the reason was found to be, that blue and yellow do not make green, but a pinkish tint, when neither prevails in the combination. It was plain, that no addition of red to this, could produce a neutral tint.

This result of mixing blue and yellow was, I believe, not previously known. It directly contradicted the received theory of colours, and seemed to be at variance with the fact, that the same blue and yellow paint, when ground together, do make green. Several experiments were proposed by Professor Forbes, in order to eliminate the effect of motion, but he was not then able to undertake them. One of these consisted in viewing alternate stripes of blue and yellow, with a telescope out of focus. I have tried this, and find the resultant tint pink as before.* I also found that the beams of light coloured by transmission through blue and yellow glasses appeared pink, when mixed on a screen, while a beam of light, after passing through both glasses, appeared green. By the help of the theory of absorption, given by Herschel, I made out the complete explanation

of this phenomenon. Those of pigments were, I think, first explained by Helm-Holtz in the manner above referred to.*

It may still be asked, whether the effect of successive presentation to the eye is identical with that of simultaneous presentation, for if there is any action of the one kind of light on the other, it can take place only in the case of simultaneous presentation. An experiment tending to settle this point is recorded by Newton (Book I. Part II., Exp. 10). He used a comb with large teeth to intercept various rays of the spectrum. When it was moved slowly, the various colours could be perceived, but when the speed was increased the result was perfect whiteness. For another form of this experiment, see Newton's Sixth Letter to Oldenburg (Horsley's Edition, vol. iv., page 335.)

In order more fully to satisfy myself on this subject, I took a disc in which were cut a number of slits, so as to divide it into spokes. In a plane, nearly passing through the axis of this disc, I placed a blue glass, so that one half of the disc might be seen by transmitted light—blue, and the other by reflected light—white. In the course of the reflected light I placed a yellow glass, and in this way I had two nearly coincident images of the slits, one yellow and one blue. By turning the disc slowly, I observed that in some parts the yellow slits and the blue slits appeared to pass over the field alternately, while in others they appeared superimposed, so as to produce alternately their mixture, which was pale pink, and complete darkness. As long as the disc moved slowly I could perceive this, but when the speed became great, the whole field appeared uniformly coloured pink, so that those parts in which the colours were seen successively were indistinguishable from those in which they were presented together to the eye.

Another form in which the experiment may be tried requires only the colour-top above described. The disc should be covered with alternate sectors of any two colours, say red and green, disposed alternately in four quadrants. By placing a piece of glass above the top, in the plane of the axis, we make the image of one half seen by reflection coincide with that of the other seen by transmission. It will then be seen that, in the diameters of the top which are parallel and perpendicular to the plane of reflection, the transmitted green coincides with the reflected green, and the transmitted red with the reflected red, so that the result is always either pure red or pure green. But in the diameters intermediate to these, the transmitted red coincides with the reflected green, and vice versa, so that the pure colours are never seen, but only their mixtures. As long as the top is spun slowly, these parts of the disc will appear more steady in colour than those in which the greatest alternations take place; but when the speed is sufficiently

^{*} I have lately seen a passage in Moigno's Cosmos, stating that M. Plateau, in 1819, had obtained gray by whirling together gamboge and Prussian blue.—Correspondence Math. et Phys., de M. Quetelet, vol. v., p. 221.

increased, the disc appears perfectly uniform in colour. From these experiments it appears, that the apparent mixture of colours is not due to a mechanical superposition of vibrations, or to any mutual action of the mixed rays, but to some cause residing in the constitution of the apparatus of vision.

(7.) Presentation of the Colours to be mixed one to each Eye.

This method is said not to succeed with some people; but I have always found that the mixture of colours was perfect, although it was difficult to conceive the objects seen by the two eyes as identical. In using the spectacles, of which one eye is green and the other red, I have found, when looking at an arrangement of green and red papers, that some looked metallic and others transparent. This arises from the very different relations of brightness of the two colours as seen by each eye through the spectacles, which suggests the false conclusion, that these differences are the result of reflection from a polished surface, or of light transmitted through a clear one.

NOTE II.

Results of Experiments with Mr HAY'S Papers, at Cambridge, November 1854.

The mean of ten observations made by six observers gave—

$$\cdot 449 \text{ R} + \cdot 299 \text{ G} + \cdot 252 \text{ B} = \cdot 224 \text{ W} + \cdot 776 \text{ Bk}$$
 . . . (1).

$$\cdot 696 \text{ R} + \cdot 304 \text{ G} = \cdot 181 \text{ B} + \cdot 327 \text{ Y} + \cdot 492 \text{ Bk}$$
 . . . (2).

These two equations served to determine the positions of white and yellow in diagram No. 2. The coefficient of W is 4.447, and that of yellow 2.506.

From these data we may deduce three other equations, either by calculation, or by measurement on the diagram (No. 2).

Eliminating green from the equations, we find—

$$\cdot 565 \text{ B} + \cdot 435 \text{ Y} = \cdot 307 \text{ R} + \cdot 304 \text{ W} + \cdot 389 \text{ Bk}$$
 . . . (3).

The mean of three observations by three different observers, gives—

$$\cdot 573 \text{ B} + \cdot 477 \text{ Y} = \cdot 313 \text{ R} + \cdot 297 \text{ W} + \cdot 390 \text{ Bk}$$

Errors of calculation, -008 B + 008 Y - 006 R + 007 W - 001 Bk

The point on the diagram to which this equation corresponds is the intersection of the lines BY and RW, and the resultant tint is a pinkish-gray.

Eliminating red from the equations, we obtain—

Calculation,
$$.533 \text{ B} + .150 \text{ G} + .317 \text{ Y} = .337 \text{ W} + .663 \text{ Bk}$$

By 10 observations, $.537 \text{ B} + .146 \text{ G} + .317 \text{ Y} = .337 \text{ W} + .663 \text{ Bk}$
Errors, $... -.004 + .004 -... -...$ (4).

Eliminating blue,
$$660 \text{ R} + 340 \text{ G} = 218 \text{ Y} + 108 \text{ W} + 682 \text{ Bk}$$

By 5 observations, $672 \text{ R} + 328 \text{ G} = 224 \text{ Y} + 094 \text{ W} + 672 \text{ Bk}$
Errors, $-012 + 012 - 006 + 014 + 008$ (5).

NOTE III.

On the Theory of Compound Colours.

Newton's theorem on the mixture of colours is to be found in his Opticks, Book I., Pt. II., Prop. VI.

In a mixture of primary colours, the quantity and quality of each being given, to know the colour of the compound.

He divides the circumference of a circle into parts proportional to the seven musical intervals, in accordance with his opinion of the divisions of the spectrum. He then conceives the colours of the spectrum arranged round the circle, and at the centre of gravity of each of the seven arcs he places a little circle, the area of which represents the number of rays of the corresponding colour which enter into the given mixture. He takes the centre of gravity of all these circles to represent the colour formed by the mixture. The hue is determined by drawing a line through the centre of the circle and this point to the circumference. The position of this line points out the colour of the spectrum which the mixture most resembles, and the distance of the resultant tint from the centre determines the fulness of its colour.

Newton, by this construction (for which he gives no reasons), plainly shows that he considered it possible to find a place within his circle for every possible colour, and that the entire nature of any compound colour may be known from its place in the circle. It will be seen that the same colour may be compounded from the colours of the spectrum in an infinite variety of ways. The apparent identity of all these mixtures, which are optically different, as may be shown by the prism, implies some law of vision not explicitly stated by Newton. This law, if Newton's method be true, must be that which I have endeavoured to establish, namely, the threefold nature of sensible colour.

With respect to Newton's construction, we now know that the proportions of the colours of the spectrum vary with the nature of the refracting medium. The only absolute index of the kind of light is the time of its vibration. The length of its vibration depends on the medium in which it is; and if any proportions are to be sought among the wave-lengths of the colours, they must be determined for those tissues of the eye in which their physical effects are supposed to terminate. It may be remarked, that the apparent colour of the spectrum changes most rapidly at three points, which lie respectively in the yellow, between blue and green, and between violet and blue. The wave-lengths of the corresponding rays in water are in the proportions of three geometric means between 1 and 2 very nearly. This result, however, is not to be considered established, unless confirmed by better observations than mine.

The only safe method of completing Newton's construction is by an examina-

tion of the colours of the spectrum and their mixtures, and subsequent calculation by the method used in the experiments with coloured papers. In this way I hope to determine the relative positions in the colour-diagram of every ray of the spectrum, and its relative intensity in the solar light. The spectrum will then form a curve not necessarily circular or even re-entrant, and its peculiarities so ascertained may form the foundation of a more complete theory of the colour-sensation.

On the relation of the pure rays of the Spectrum to the three assumed Elementary Sensations.

If we place the three elementary colour-sensations (which we may call, after Young, red, green, and violet) at the angles of a triangle, all colours which the eye can possibly perceive (whether by the action of light, or by pressure, disease, or imagination), must be somewhere within this triangle, those which lie farthest from the centre being the fullest and purest colours. Hence the colours which lie at the middle of the sides are the purest of their kind which the eye can see, although not so pure as the elementary sensations.

It is natural to suppose that the pure red, green, and violet rays of the spectrum produce the sensations which bear their names in the highest purity. But from this supposition it would follow that the yellow, composed of the red and green of the spectrum, would be the most intense yellow possible, while it is the result of experiment, that the yellow of the spectrum itself is much more full in colour. Hence the sensations produced by the pure red and green rays of the spectrum are not the pure sensations of our theory. Newton has remarked, that no two colours of the spectrum produce, when mixed, a colour equal in fulness to the intermediate colour. The colours of the spectrum are all more intense than any compound ones. Purple is the only colour which must be produced by combination. The experiments of Helmholtz lead to the same conclusion; and hence it would appear that we can find no part of the spectrum which produces a pure sensation.

An additional, though less satisfactory evidence of this, is supplied by the observation of the colours of the spectrum when excessively bright. They then appear to lose their peculiar colour, and to merge into pure whiteness. This is probably due to the want of capacity of the organ to take in so strong an impression; one sensation becomes first saturated, and the other two speedily follow it, the final effect being simple brightness.

From these facts I would conclude, that every ray of the spectrum is capable of producing all three pure sensations, though in different degrees. The curve, therefore, which we have supposed to represent the spectrum will be quite within the triangle of colour. All natural or artificial colours, being compounded of the colours of the spectrum, must lie within this curve, and, therefore, the colours corresponding to those parts of the triangle beyond this curve must be for ever

unknown to us. The determination of the exact nature of the pure sensations, or of their relation to ordinary colours, is therefore impossible, unless we can prevent them from interfering with each other as they do. It may be possible to experience sensations more pure than those directly produced by the spectrum, by first exhausting the sensibility to one colour by protracted gazing, and then suddenly turning to its opposite. But if, as I suspect, colour-blindness be due to the absence of one of these sensations, then the point D in diagram (2), which indicates their absent sensation, indicates also our pure sensation, which we may call red, but which we can never experience, because all kinds of light excite the other sensations.

Newton has stated one objection to his theory, as follows:—" Also, if only two of the primary colours, which in the circle are opposite to one another, be mixed in an equal proportion, the point Z" (the resultant tint) "shall fall upon the centre O" (neutral tint); " and yet the colour compounded of these two shall not be perfectly white, but some faint anonymous colour. For I could never yet, by mixing only two primary colours, produce a perfect white." This is confirmed by the experiments of Helmholtz; who, however has succeeded better with some pairs of colours than with others.

In my experiments on the spectrum, I came to the same result; but it appeared to me that the very peculiar appearance of the neutral tints produced was owing to some optical effect taking place in the transparent part of the eye on the mixture of two rays of very different refrangibility. Most eyes are by no means achromatic, so that the images of objects illuminated with mixed light of this kind appear divided into two different colours; and even when there is no distinct object, the mixtures become in some degree analysed, so as to present a very strange, and certainly "anonymous" appearance.

Additional Note on the more recent experiments of M. Helmholtz.*

In his former memoir on the Theory of Compound Colours, † M. Helmholtz arrived at the conclusion that only one pair of homogeneous colours, orange-yellow and indigo-blue, were strictly complementary. This result was shown by Professor Grassmann; to be at variance with Newton's theory of compound colours; and although the reasoning was founded on intuitive rather than experimental truths, it pointed out the tests by which Newton's theory must be verified or overthrown. In applying these tests, M. Helmholtz made use of an apparatus similar to that described by M. Foucault, § by which a screen of white paper is illuminated by the mixed light. The field of mixed colour is much larger than

^{*} Poggendorff's Annulen, Bd. xciv. (I am indebted for the perusal of this Memoir to Professor STOKES.)

[†] Ib. Bd. lxxxvii. Annals of Philosophy, 1852, Part II. ‡ Ib. Bd. lxxxix., Ann. Phil., 1854, April. § Ib. Bd. lxxxviii. Moigno, Cosmos, 1853, Tom. ii., p. 232.

in M. Helmholtz's former experiments, and the facility of forming combinations is much increased. In this memoir the mathematical theory of Newton's circle, and of the curve formed by the spectrum, with its possible transformations, is completely stated, and the form of this curve is in some degree indicated, as far as the determination of the colours which lie on opposite sides of white, and of those which lie opposite the part of the curve which is wanting. The colours between red and yellow-green are complementary to colours between blue-green and violet, and those between yellow-green and blue-green have no homogeneous complementaries, but must be neutralized by various hues of purple, i.e., mixtures of red and violet. The names of the complementary colours, with their wavelengths in air, as deduced from Fraunhofer's measurements, are given in the following table:—

Colour.	Wave-length.	Complementary Colour.	Wave-length.	Ratio of Wave-lengths.
Red,	2425	Green-blue, .	1818	1:334
Orange,	2244	Blue,	1809	1.240
Gold-yellow,	2162	Blue,	1793	1.206
Gold-yellow,	2120	Blue,	1781	1.190
Yellow,	2095	Indigo-blue, .	1716	1.221
Yellow,	2085	Indigo-blue, .	1706	1.222
Green-yellow,	2082	Violet,	1600 —	1.301

(In order to reduce these wave-lengths to their actual length in the eye, each must be divided by the index of refraction for that kind of light in the medium in which the physical effect of the vibrations is supposed to take place.)

Although these experiments are not in themselves sufficient to give the complete theory of the curve of homogeneous colours, they determine the most important element of that theory in a way which seems very accurate, and I cannot doubt that when a philosopher who has so fully pointed out the importance of general theories in physics turns his attention to the theory of sensation, he will at least establish the principle that the laws of sensation can be successfully investigated only after the corresponding physical laws have been ascertained, and that the connection of these two kinds of laws can be apprehended only when the distinction between them is fully recognised.

NOTE IV.

Description of the Figures. Plate VI.

- No. 1. is the colour-diagram already referred to, representing, on Newton's principle, the relations of different coloured papers to the three standard colours—vermilion, emerald-green, and ultramarine. The initials denoting the colours are explained in the list at page 276, and the numbers belonging to them are their coefficients of intensity, the use of which has been explained. The initials H.R., H.B., and H.G., represent the red, blue and green papers of Mr HAY, and serve to connect this diagram with No. (2), which takes these colours for its standards.
- No. 2. represents the relations of Mr Hav's red, blue, green, white, and yellow papers, as determined by a large number of experiments at Cambridge.—(See Note II.). The use of the point D, in calculating the results of colour-blindness, is explained in the Paper.
- Fig. 3. represents a disc of the larger size, with its slit.
- Fig. 4. shows the mode of combining two discs of the smaller size.
- Fig. 5. shows the combination of discs, as placed on the top, in the first experiment described in the Paper.
- Fig. 6. represents the method of spinning the top, when speed is required.

 The last four figures are half the actual size.
- Colour-tops of the kind used in these experiments, with paper discs of the colours whose relations are represented in No. 1, are to be had of Mr J. M. Bryson, Opticisu, Edinburgh.

